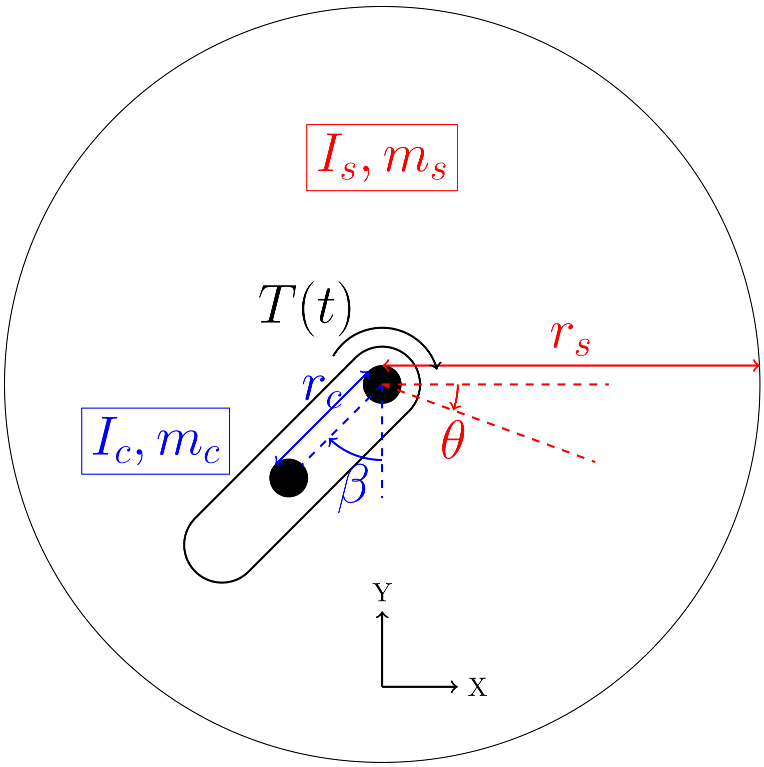
Balancing Spool Diagram



I_s, m_s = mass and inertia of spool
I_c, m_c = mass and inertia of counterweight
r_c = radius from COM spool to COM counterweight
r_s = radius from COM spool to outer edge
θ = angle of spool (CW+)
β = angle of counterweight (CW+)
$T(t)$ = torque from motor on center of spool

$$\frac{d}{dt}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & \frac{-b(I_c+m_c r_c^2)}{(m_c r_c^2(I_s+m_s r_s^2)+I_c(I_s+(m_c+m_s)r_s^2)} & \frac{g m_c^2 r_c^2 r_s}{(m_c r_c^2(I_s+m_s r_s^2)+I_c(I_s+(m_c+m_s)r_s^2)} & \frac{-c m_c r_c r_s}{(m_c r_c^2(I_s+m_s r_s^2)+I_c(I_s+(m_c+m_s)r_s^2)} \\ 0 & 0 & 0 & 1 \\ 0 & \frac{b m_c r_c r_s}{(m_c r_c^2(I_s+m_s r_s^2)+I_c(I_s+(m_c+m_s)r_s^2)} & \frac{-g m_c r_c (I_s+(m_c+m_s)r_s^2)}{(m_c r_c^2(I_s+m_s r_s^2)+I_c(I_s+(m_c+m_s)r_s^2)} & \frac{c(m_c+m_s)r_s^2}{(m_c r_c^2(I_s+m_s r_s^2)+I_c(I_s+(m_c+m_s)r_s^2)} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{I_c+m_c r_c^2}{(m_c r_c^2(I_s+m_s r_s^2)+I_c(I_s+(m_c+m_s)r_s^2)} \\ 0 \\ \frac{m_c r_c r_s}{(m_c r_c^2(I_s+m_s r_s^2)+I_c(I_s+(m_c+m_s)r_s^2)} \end{bmatrix} T(t)$$

$$Y(t) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$